Engineering Notes

Efficiency of Waffle Grid Panels under Compressive Loading

George S. Johnston* and Robert B. Lantz†
Lockheed-Georgia Company, Marietta, Georgia

Nomenclature

a = waffle plate width, in.
b_s = grid spacing, in.
b_w = stiffener height, in.
C_s = stress coefficient
D = flexural stiffness, lb/in.
E = modulus of elasticity of skin and stiffeners, psi
k_s = buckling coefficient of skin
k_w = buckling coefficient of web
P_i = loading per unit width, lb/in.
t_s = skin thickness, in.
t_w = stiffener thickness, in.
η = plasticity coefficient
ν = Poisson's ratio
σ = actual stress, psi
σ' = effective stress, psi
σ_{cr} = critical buckling stress, psi

Introduction

References 1–4 are prominent articles on waffle grid panels. Schmidt and Kicher[1] considered synthesis of finite panels under combined loads on an initial buckling basis. The panels were considered to be simply supported at their boundary. Local buckling of the plate elements making up the waffle grid was based on the simple support idealization, whereas the aspect ratio of the stiffener elements was not taken into account. Gerard[2] previously used this same idealization in determining the efficiency of long waffle grid panels supported at the edges and loaded in compression (same boundary conditions as presently considered). Gerard also speculated that the efficiency of 45° waffle grid would be improved over that for 0° and 90° waffle grid by the same factor as that demonstrated by Dow,[3] who analyzed and tested shallow waffle grid (like practical chem-milled configurations). Emero and Spunt[4] (whose paper became available after this note was originally submitted) took the aspect ratio of the stiffeners into account but not the rotational restraint (which is the consideration added herein). They also considered rectangular panels and proved that the 45° waffle grid panels are inferior to the 0° and 90° waffle grid on an elastic buckling basis. The crooked load path in 45° waffle grid panels causes higher internal stresses so that its attainable stress is limited to a lower level because of yielding. Only 0° and 90° symmetrical waffle grid panels are considered herein.

Local Buckling of Waffle Grid

The buckling mode that the skin would take if it were simply supported interferes with the mode that the stiffeners would take if they were simply supported. The lowest order buckling stress with the skin as the critical element involves bending of the stiffeners as though they were fixed at their intersections as shown in Fig. 1. The lowest buckling stress with the stiffeners as the unstable element corresponds to bending of the skin in an unusual mode so that opposite rotation of adjacent stiffeners is accommodated.

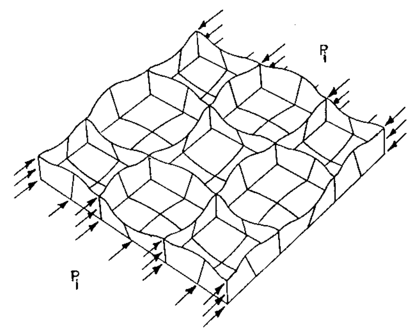

Fig. 1 Buckling mode.

The buckling stresses are estimated rather than determined by accurate evaluation of theory. If the skin were simply supported, the buckling coefficient k_s would be 4.0. If clamped, $k_s = 10.3$ (Ref. 5). The mode shown in Fig. 1 is estimated to occur at $k_s = 6.0$ so that with $\nu = 0.316$

$$\sigma_{cr} = 5.45\, E(t_s/b_s)^2 \tag{1}$$

If the skin were subject to the loading corresponding to buckling with clamped edges, it would offer no resistance to buckling of the stiffeners in their natural mode (which involves considerable end restraint because of the unloaded members at 90° to the load). The buckling stress of columns with similar restraint is given by Timoshenko.[6] The critical stress is increased over that for a simply supported column by the factor $\frac{5}{3}$. Applying this factor to the length term of the buckling coefficient for a simply supported plate (p. 340, Ref. 6) yields $k_w = 0.456 + \frac{5}{3}\,(b_w/b_s)^2$ or

$$\sigma_{cr} = [0.4142 + 1.514\,(b_w/b_s)^2]E\,(t_w/b_w)^2 \tag{2}$$

where b_w is the stiffener height, and b_s is the width of the (square) skin element.

The stiffness of the waffle grid is

$$D = (Eb_w^3\, t_w/12b_s)(4b_st_s + b_wt_w)/(b_st_s + b_wt_w) \tag{3}$$

which is the product of the elastic modulus E and the moment of inertia per unit width, omitting the moment of inertia of the skin about its own axis. This is the same result as obtained in Ref. 2 and utilized in Ref. 4 except that the plate stiffness factor $1/(1 - \nu^2)$ should be left out, since the stiffeners constitute the main portion of the inertia and are stressed uniaxially.

The loading for over-all buckling of the waffle panel supported at the unloaded edges is conservatively given by

$$P_i = 2\pi^2\, D/a^2 \tag{4}$$

which omits the torsional stiffness. The plate width is a.

The attainable value of the effective stress is given by

$$\sigma' = C_s(\eta EP_i/a)^{1/2} \tag{5}$$

where C_s is the stress coefficient. It is proportional to the attainable stress and analogous to the lift coefficient in aerodynamics. The effective stress accounts for the material in

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* Development Engineer Specialist, Analysis Methods and Procedures Department. Member AIAA.
† Aircraft Structures Engineer, Analysis Methods and Procedures Department. Associate Member AIAA.

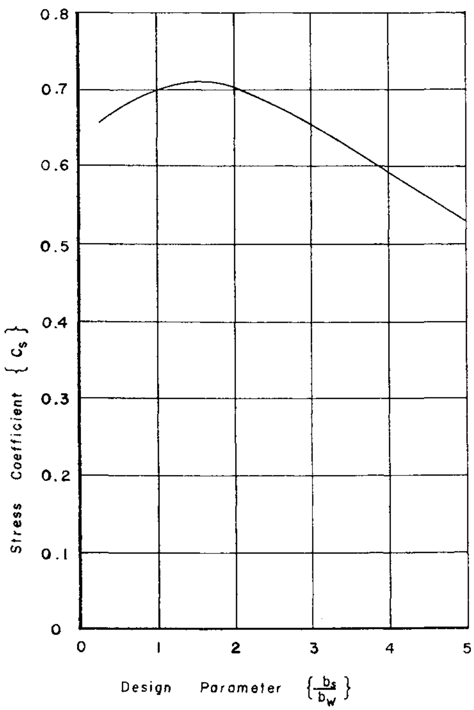

Fig. 2 Results, stress coefficient (C_s) vs design parameter (b_s/b_w).

the transverse stiffeners

$$\sigma' = \sigma(b_s t_s + b_w t_w)/(b_s t_s + 2b_w t_w) \quad (6)$$

The optimum design corresponds to the panel being critical for local and general buckling at the same stress level. Since both the skin and the stiffeners are critical for local buckling, this still leaves one dimensional ratio (b_s/b_w) to be determined. Values of C_s vs b_s/b_w are plotted in Fig. 2. These were calculated by applying the equations in the following order: assume b_s/b_w and regard σ and E as known, solve Eq. (1) for b_s/t_s, solve Eq. (2) for b_w/t_w, calculate loading P_i from stress multiplied by area, substitute P_i and Eq. (3) into Eq. (4) to solve for a, and substitute Eq. (6) into Eq. (5) to solve for the stress coefficient C_s.

Table 1 C_s values for symmetrical, edge supported, 0° and 90° waffle panels

C_s	Ref.	Assumptions
0.60	2	Simple support, long stiffener plate elements
0.66	4	Simple support, finite stiffener plate elements
0.71	This note	Estimated rotational restraint, finite stiffener plate elements

Conclusions

The maximum values of C_s for symmetrical, edge supported, 0° and 90° waffle panels are given in Table 1. Waffle grid does not compare favorably with truss core sandwich, which has a stress coefficient $C_s = 1.05$ (from Ref. 7).

References

[1] Schmidt, L. A. and Kicher, T. P., "Structural synthesis of symmetric waffle plate," NASA TN D1691 (December 1962).

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[6] Timoshenko, S., *Theory of Elastic Stability* (McGraw-Hill Book Co., Inc., New York, 1936), pp. 91, 318, and 340.

[7] Crawford, R. F. and Bivins, A. B., "Minimum weight potential and design information for stiffened plates and shells," ARS Preprint 2424–62, p. 42 (February 1962); also AIAA J. 1, 879–886 (1963).

Editor's Note

Beginning with this issue, marine systems papers will be grouped in a Marine Systems Supplement (see the following pages). We feel that the basic principles of marine vehicle engineering and operation are similar to those which apply to aircraft and that the *Journal of Aircraft,* therefore, is the proper medium for presentation of such information. The Marine Systems Supplement is being established to facilitate identification of articles in this special field of interest.